Probabilities of Default for Impairment Under IFRS 9

Antoine Conze
Hiram Finance
antoine.conze@hiram-finance.com

November 2015

1 Introduction

Probabilities of default built for regulatory purposes cannot be applied directly to expected credit losses impairment calculations under the IFRS 9 new standard. This is because the regulatory framework requires stressed through-the-cycle (TTC) probabilities, so as to avoid a procyclical capital charge calculation, while IFRS 9 requires point-in-time (PIT) probabilities that include forward looking information.

However the model (Asymptotic Single Risk Factor model or similar) that is generally used for the construction of stressed TTC probabilities of default can still be used to build forward looking PIT probabilities of default, provided it is made explanatory so as to obtain probabilities that are conditional on current and forecast economic conditions.

This note explores how this can be achieved, thus leveraging on developments that institutions have already made in order to comply with regulatory requirements.

2 Notations

Let $i = 1, ..., K$ be a rating system with $i = 1$ the best rating and $i = K$ the default. We observe every year end $t = 1, ..., T$ a point-in-time rating migration frequency matrix $(f_{i,j,t})$ where $f_{i,j,t}$ is the proportion of i-rated entities that have moved to rating j by year end. We also note $f^d_{i,t} = \sum_{j=i+1}^{K} f_{i,j,t}$ the downgrading frequency (transition from rating i to rating $i + 1$ or worse).

We also denote $\widehat{E}[x_t] = \sum_{t=1}^{T} x_t/T$ and $\widehat{\mathrm{VAR}}[x_t] = \sum_{t=1}^{T} x_t^2/T - \widehat{E}[x_t]^2$ respectively the empirical mean and variance of a time series x_t, and $\widehat{\mathrm{COV}}[x_t, y_t] = \sum_{t=1}^{T} x_t y_t/T - \widehat{E}[x_t]\widehat{E}[y_t]$ the empirical covariance between two time series x_t and y_t.

3 ASRF Model

The version of the Asymptotic Single Risk Factor Model (ASFR) model we use here is slightly generalized so as to make the correlation between same rating entities dependent on the rating, following the specification proposed by Yang and Du [5]. Different specifications can be used, which would lead to slightly different versions of all the formulas that follow.

We assume that entities with same rating are homogeneous and that for an entity n with rating i migration to rating j occurs if a latent variable $z_{n,i,t}$, interpreted as the normalized return of the entity latent asset, takes a value comprised between the transition thresholds $b_{i,j}$ and $b_{i,j-1}$, with the convention $b_{i,0} = +\infty$ and $b_{i,K} = -\infty$.

Transition probabilities: the transition probability from rating i to rating j is

$$p_{i,j} = P(b_{i,j} < z_{n,i,t} \leq b_{i,j-1}) = \Phi\left(b_{i,j-1}\right) - \Phi\left(b_{i,j}\right)$$

where Φ is the Gaussian cumulative distribution function, the probability of default is

$$p_{i,K} = \Phi\left(b_{i,K-1}\right)$$

and the probability of downgrading is

$$p_i^d = \sum_{j=i+1}^{K} p_{i,j} = \Phi\left(b_{i,i}\right).$$

Conditional transition probabilities: we assume that $z_{n,i,t}$ splits into $z_{n,i,t} = \sqrt{\rho_i} s_{i,t} + \sqrt{1-\rho_i}\epsilon_{n,t}$ where $s_{i,t}$ is the systemic risk common to all entities with rating i and $\epsilon_{n,t}$ is the entity idiosyncratic risk. The $s_{i,t}$ and $\epsilon_{n,t}$ are independent and $\sim N(0,1)$. Here ρ_i is the correlation between entities with same rating i. The transition probabilities conditional on $s_{i,t}$ are

$$p_{i,j}(s_{i,t}) = P(b_{i,j} < z_{n,i,t} \leq b_{i,j-1}|s_{i,t}) = \Phi\left(\frac{b_{i,j-1} - \sqrt{\rho_i} s_{i,t}}{\sqrt{1-\rho_i}}\right) - \Phi\left(\frac{b_{i,j} - \sqrt{\rho_i} s_{i,t}}{\sqrt{1-\rho_i}}\right)$$

the conditional probability of default is

$$p_{i,K}(s_{i,t}) = \Phi\left(\frac{b_{i,K} - \sqrt{\rho_i} s_{i,t}}{\sqrt{1-\rho_i}}\right)$$

and the conditional probability of downgrading is

$$p_i^d(s_{i,t}) = \sum_{j=i+1}^{K} p_{i,j}(s_{i,t}) = \Phi\left(\frac{b_{i,i} - \sqrt{\rho_i} s_{i,t}}{\sqrt{1-\rho_i}}\right).$$

Estimation of the transition thresholds $b_{i,j}$: since

$$\sum_{k=j+1}^{K} p_{i,k} = \Phi\left(b_{i,j}\right)$$

the transition thresholds are usually estimated from the empirical mean of point-in-time transition matrices

$$b_{i,j} = \Phi^{-1}\left(\sum_{k=j+1}^{K} \widehat{E}[f_{i,k,t}]\right).$$

In particular we have for the downgrading threshold

$$b_{i,i} = \Phi^{-1}\left(\widehat{E}[f_{i,t}^d]\right).$$

Note that this approach is in principle valid only for asymptotically large pools. With finite pools an alternative is to jointly estimate the $b_{i,j}$ and the ρ_i from the conditional probabilities $p_{i,j}(s_{i,t})$ and a binomial maximum likelihood procedure, as in the next section. However common practice is to use the above formulas, with the mean matrix $\widehat{f}_{i,j} = \widehat{E}[f_{i,j,t}]$ possibly subjected to an expert review, so as to satisfy

- $\widehat{f}_{i,K-1}$ is an increasing function of i (a better rating has a lower probability of default);
- $\widehat{f}_{i,i+k}$ and $\widehat{f}_{i,i-k}$ are decreasing functions of k (the probability of migrating to a nearby rating is greater than the probability of migrating to a far away rating).

Estimation of the correlations ρ_i: in Merton (see [3]) type models the ρ_i are often estimated from issuers stock prices correlations, but in the credit world it is more common to estimate correlations from the $f_{i,j,t}$, with a procedure such as proposed in Demey and al [4]. Here we follow the idea proposed in Yang and Du [5] of working with downgrading frequencies $f^d_{i,t}$ rather than with default frequencies $f_{i,K,t}$, which is motivated by the fact that default frequencies for good quality ratings are very low.

Estimation on a finite pool: given the downgrading probability $p^d_i(s_{i,t})$ the likelihood of observing $k_{i,t} = f^d_{i,t} n_{i,t}$ downgrades among $n_{i,t}$ entities with rating i is

$$\frac{n_{i,t}!}{k_{i,t}!(n_{i,t}-k_{i,t})!}\left(p^d_{i,t}\right)^{k_{i,t}}\left(1-p^d_{i,t}\right)^{(n_{i,t}-k_{i,t})}.$$

The unconditional likelihood computed by taking the expectation against the distribution of $s_{i,t}$ is

$$\begin{aligned} L_{i,t}(\rho_i) &= E\left[\frac{n_{i,t}!}{k_{i,t}!(n_{i,t}-k_{i,t})!}\left(p^d_{i,t}\right)^{k_{i,t}}\left(1-p^d_{i,t}\right)^{(n_{i,t}-k_{i,t})}\right] \\ &= \int_{-\infty}^{+\infty}\frac{n_{i,t}!}{k_{i,t}!(n_{i,t}-k_{i,t})!}\left(\Phi\left(\frac{b_{i,i}-\sqrt{\rho_i}s}{\sqrt{1-\rho_i}}\right)\right)^{k_{i,t}}\left(1-\Phi\left(\frac{b_{i,i}-\sqrt{\rho_i}s}{\sqrt{1-\rho_i}}\right)\right)^{(n_{i,t}-k_{i,t})}\varphi(s)ds \end{aligned}$$

where φ is the Gaussian density. The last part is to minimize the negative log-likelihood

$$-\ln L(\rho_i) = -\sum_{t=1}^{T}\ln\left(L_{i,t}(\rho_i)\right)$$

to obtain an estimate of the ρ_i.

Estimation on an asymptotic pool of infinite size: $p^d_i(s_{i,t})$ can be directly identified to $f^d_{i,t}$, or equivalently $\dfrac{b_{i,i}-\sqrt{\rho_i}s_{i,t}}{\sqrt{1-\rho_i}}$ can be identified to $\Phi^{-1}\left(f^d_{i,t}\right)$. The negative log-likelihood is (with $a_{1,i} = -\sqrt{\rho_i/(1-\rho_i)}$ to simplify the exposition)

$$-\ln L(a_{1,i}) = \sum_{t=1}^{T}\frac{\left(\Phi^{-1}\left(f^d_{i,t}\right)-b_{i,i}\sqrt{1+a^2_{1,i}}\right)^2}{2a^2_{1,i}} + T\ln\left(a_{1,i}\right) - \frac{1}{2}\sum_{t=1}^{T}\Phi^{-1}\left(f^d_{i,t}\right)^2.$$

There is no explicit formula for obtaining the minimum of $-\ln L(a_{1,i})$, but a simple Newton-Raphson method applied to its derivative is sufficient to estimate $a_{1,i}$ and obtain $\rho_i = a^2_{1,i}/(1+a^2_{1,i})$.

Joint estimation of ρ_i and $b_{i,i}$: alternatively we can estimate jointly the ρ_i and the $b_{i,i}$. Let $a_{0,i} = b_{i,i}/\sqrt{1-\rho_i}$, then the conditional downgrading probability is $p^d_i(s_{i,t}) = \Phi\left(a_{0,i} + a_{1,i}s_{i,t}\right)$, onto which we can apply a maximum likelihood as above. In the asymptotic case the negative log-likelihood becomes

$$-\ln L(a_{0,i}, a_{1,i}) = \sum_{t=1}^{T}\frac{\left(\Phi^{-1}\left(f^d_{i,t}\right)-a_{0,i}\right)^2}{2a^2_{1,i}} + T\ln\left(a_{1,i}\right) - \frac{1}{2}\sum_{t=1}^{T}\Phi^{-1}\left(f^d_{i,t}\right)^2$$

and we obtain explicitly after minimization

$$\begin{aligned} a_{0,i} &= \widehat{E}\left[\Phi^{-1}\left(f^d_{i,t}\right)\right] \\ a^2_{1,i} &= \widehat{\mathrm{VAR}}\left[\Phi^{-1}\left(f^d_{i,t}\right)\right] \end{aligned}$$

Note that we have for the $b_{i,i}$

$$b_{i,i} = a_{0,i}\sqrt{1-\rho_i} = \frac{a_{0,i}}{\sqrt{1+a_{1,i}^2}} = \frac{\widehat{E}\left[\Phi^{-1}\left(f_{i,t}^d\right)\right]}{\sqrt{1+\widehat{\mathrm{VAR}}\left[\Phi^{-1}\left(f_{i,t}^d\right)\right]}}$$

to be compared with the estimate

$$b_{i,i} = \Phi^{-1}\left(\widehat{E}[f_{i,t}^d]\right)$$

obtained previously. These two estimates are consistent since

$$\Phi^{-1}\left(E[p_i^d(s_{i,t})]\right) = \Phi^{-1}\left(E\left[\Phi\left(a_{0,i}+a_{1,i}s_{i,t}\right)\right]\right) = \frac{a_{0,i}}{\sqrt{1+a_{1,i}^2}}.$$

4 Regulatory Framework and IFRS 9 Framework

In the Basel regulatory framework (see for instance [1]) the goal is to build a stressed TTC transition matrix, unconditional to the current point in the economic cycle, so as to avoid increased procyclicality of regulatory capital. The ASRF model is a natural candidate to this exercise, because to a confidence level α corresponds a value $s_{i,t} = \Phi^{-1}(\alpha)$ of the systemic risk from which we obtain stressed transition probabilities

$$p_{i,j}(\Phi^{-1}(\alpha)) = \Phi\left(\frac{b_{i,j-1}-\sqrt{\rho_i}\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}}\right) - \Phi\left(\frac{b_{i,j}-\sqrt{\rho_i}\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}}\right)$$

and for stressed probabilities of default ($j = K$)

$$p_{i,K}(\Phi^{-1}(\alpha)) = \Phi\left(\frac{b_{i,K-1}-\sqrt{\rho_i}\Phi^{-1}(\alpha)}{\sqrt{1-\rho_i}}\right)$$

On the opposite the IFRS 9 standard insist on the forward looking feature of required PIT probabilities of default as these must incorporate “general economic conditions and an assessment of both the current as well as the forecast direction of conditions at the reporting date” (see IASB [2]).

Thus the model presented up to this point is not applicable as is because it is not explanatory in the sense that $s_{i,t}$ is a latent variable. To make the model forward looking it has to be made explanatory by incorporating macroeconomic and market factors observations and forecasts, and computing probabilities of default conditional to these.

5 Explanatory Model

To build a simple explanatory model we assume that $s_{i,t}$ can be decomposed into a linear function of macroeconomic variables $x_{m,t}$ deemed relevant, plus a latent random residual.

In order to simplify notations (see the formula below for the downgrading probabilities) and with no loss of generality we write the decomposition of $s_{i,t}$ as

$$\frac{b_{i,i}-\sqrt{\rho_i}s_{i,t}}{\sqrt{1-\rho_i}} = a_{0,i} + \sum_{m=1}^{M} a_{m,i}x_{m,t} + \sigma_i\varepsilon_{i,t}$$

where the $\varepsilon_{i,t}$ are independent and $\sim N(0,1)$. We obtain for the transition probabilities conditional on the $x_{m,t}$

$$\begin{aligned} p_{i,j}(x_{1,t},...,x_{M,t}) &= E\left[p_{i,j}(s_{i,t})|x_{1,t},...,x_{M,t}\right] \\ &= \Phi\left(\frac{b_{i,j-1}-b_{i,i}}{\sqrt{(1-\rho_i)(1+\sigma_i^2)}}+\frac{a_{0,i}+\sum_{m=1}^{M}a_{m,i}x_{m,t}}{\sqrt{1+\sigma_i^2}}\right) \\ &\quad -\Phi\left(\frac{b_{i,j}-b_{i,i}}{\sqrt{(1-\rho_i)(1+\sigma_i^2)}}+\frac{a_{0,i}+\sum_{m=1}^{M}a_{m,i}x_{m,t}}{\sqrt{1+\sigma_i^2}}\right) \end{aligned}$$

for the probabilities of default ($j = K$) conditional on the $x_{m,t}$

$$p_{i,j}(x_{1,t},...,x_{M,t}) = \Phi\left(\frac{b_{i,K-1}-b_{i,i}}{\sqrt{(1-\rho_i)(1+\sigma_i^2)}}+\frac{a_{0,i}+\sum_{m=1}^{M}a_{m,i}x_{m,t}}{\sqrt{1+\sigma_i^2}}\right)$$

and for the downgrading probabilities conditional on the $x_{m,t}$

$$p_i^d(x_{1,t},...,x_{M,t}) = E\left[p_i^d(s_{i,t})|x_{1,t},...,x_{M,t}\right] = \Phi\left(\frac{a_{0,i}+\sum_{m=1}^{M}a_{m,i}x_{m,t}}{\sqrt{1+\sigma_i^2}}\right).$$

Estimation on a finite pool: the $a_{m,i}$ and σ_i coefficients are estimated in the same fashion as the ρ_i in section 3: the unconditional likelihood is computed by taking the expectation against the distribution of $e_{i,t}$ and becomes

$$\begin{aligned} L_{i,t}(a_{m,i},\sigma_i) &= \int_{-\infty}^{+\infty}\frac{n_{i,t}!}{k_{i,t}!(n_{i,t}-k_{i,t})!}\left(\Phi\left(a_{0,i}+\sum_{m=1}^{M}a_{m,i}x_{m,t}+\sigma_i e\right)\right)^{k_{i,t}} \\ &\quad \left(1-\Phi\left(a_{0,i}+\sum_{m=1}^{M}a_{m,i}x_{m,t}+\sigma_i e\right)\right)^{(n_{i,t}-k_{i,t})}\varphi(e)de \end{aligned}$$

and we have to minimize the negative log-likelihood

$$-\ln L(a_{m,i},\sigma_i) = -\sum_{t=1}^{T}\ln\left(L_{i,t}(a_{m,i},\sigma_i)\right)$$

to obtain an estimate of the $a_{m,i}$ and σ_i.

Estimation on an asymptotic pool of infinite size: as in section 3 we identify $p_i^d(s_{i,t})$ and $f_{i,t}^d$. The negative log-likelihood becomes

$$-\ln L(a_{m,i},\sigma_i) = \sum_{t=1}^{T}\frac{\left(\Phi^{-1}\left(f_{i,t}^d\right)-a_{0,i}-\sum_{m=1}^{M}a_{m,i}x_{m,t}\right)^2}{2\sigma_i^2}+T\ln(\sigma_i)-\frac{1}{2}\sum_{t=1}^{T}\Phi^{-1}\left(f_{i,t}^d\right)^2$$

and we obtain explicitly after minimization

$$\begin{aligned} a_{m,i} &= \widehat{\mathrm{COV}}\left[\Phi^{-1}\left(f_{i,t}^d\right),x_{m,t}\right]/\widehat{\mathrm{VAR}}\left[x_{m,t}\right] \\ a_{0,i} &= \widehat{E}\left[\Phi^{-1}\left(f_{i,t}^d\right)\right]-\sum_{m=1}^{M}a_{m,i}\widehat{E}\left[x_{m,t}\right] \\ \sigma_i^2 &= \widehat{\mathrm{VAR}}\left[\Phi^{-1}\left(f_{i,t}^d\right)-a_{0,i}-\sum_{m=1}^{M}a_{m,i}x_{m,t}\right]. \end{aligned}$$

6 IFRS 9 PIT probabilities of default

From the explanatory model in the previous section and observations or forecast of the $x_{m,t}$ variables we build the PIT transition probabilities matrices $p_{i,j}(x_{1,t},...,x_{M,t})$ that can be used in the IFRS 9 framework to assess the change in credit quality and compute the expected credit losses.

Note that the $p_{i,j}(x_{1,t},...,x_{M,t})$ probabilities are for a 1 year horizon. The IFRS 9 standard requires lifetime probabilities of default for every instrument to assess their possible change in credit quality, hence probabilities for any horizon T conditional on the economic forecast $(x_{1,h},...,x_{M,h})$, $h = t,...,T$. These are readily obtained by taking the product of the corresponding 1 year conditional transition matrices (extended with $p_{K,j}(x_{1,h},...,x_{M,h}) = 0$, $j < K$, and $p_{K,K}(x_{1,h},...,x_{M,h}) = 1$ to obtain square matrices that reflect the absorbing nature of the default state)

$$\prod_{h=t}^{T} \left(p_{i,j}(x_{1,h},...,x_{M,h})\right).$$

The last point is to deal with the uncertainties that pertain to economic forecast of the $x_{m,t}$ variables. The IFRS 9 standard specifies that calculations should reflect "an unbiased and probability weighted amount that is determined by evaluating a range of possible outcomes" (see IASB [2]). In essence this means that the forecast for the $x_{m,t}$ should probably include a stochastic modeling part. This can be achieved through a specification of stochastic processes for the $x_{m,t}$, such as autoregressive processes, calibrated both on historical data and economic forecast. The corresponding T-horizon transition and default probabilities will then be obtained as

$$E\left[\prod_{h=t}^{T} \left(p_{i,j}(x_{1,h},...,x_{M,h})\right)\right].$$

which can be computed using a Monte Carlo procedure.

7 Conclusion

Institutions will want to implement the IFRS 9 standard by leveraging on developments that have been made to comply with regulatory requirements. This note explores how to modify the models that are generally used for the construction of regulatory stressed TTC probabilities of default, so as to obtain forward looking PIT probabilities that can be used within the IFRS 9 framework.

References

[1] Basel Committee. (2004). An Explanatory Note on the Basel II IRB Risk Weight Functions.

[2] IASB (2014). International Financial Reporting Standard 9 Financial Instruments. Including Implementation Guidance and Basis for Conclusions (IFRS 9, final standard)

[3] Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. The Journal of Finance, 29(2), 449-470.

[4] Demey, P., Jouanin, J. F., Roget, C., & Roncalli, T. (2004). Maximum likelihood estimate of default correlations. Risk Magazine November 2004.

[5] Yang, B. H., & Du, Z. (2015). Stress Testing and Modeling of Rating Migration under the Vasicek Model Framework-Empirical approaches and technical implementation, Journal of Risk Model Validation (9)